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RESEARCH DEPARTMENT

SATELLITE BROADCASTING SERVICE AREAS

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(RA-53)

Research Department Report No. **1970/3**

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SATELLITE BROADCASTING SERVICE AREAS

SUMMARY

The coverage area provided by a transmitter on a geostationary satellite emitting a right-circular conical beam is mainly considered. For sufficiently small beamwidth, the service area is nearly an ellipse in the tangent plane to the Earth at the target point, and the axial lengths and directions are calculated. In general, the service-area boundary is best specified by plotting latitude against longitude, and formulae for specifying this boundary for arbitrary beamwidth are given and discussed. In particular, the 'tulip' shape of some of the diagrams of Fig. 2 (reproduced from reference 1) is explained.

1. INTRODUCTION

If a satellite is to be geostationary, it must be above a point on the Equator, and at a distance of about 42,200 km from the centre of the Earth. The beam emanating from the satellite will generally be assumed to be a right-circular cone (the case of a general conical beam is discussed in outline in Section 5). This cone will meet the Earth's surface in a curve, as illustrated in Fig. 1. For sufficiently small beamwidth, the curve is approximately an ellipse in the tangent plane to the Earth at the target point. For larger beamwidth, the curve will cease to lie in one plane, although it will remain a closed curve unless the beamwidth is so great that part of the cone of radiation from the satellite lies outside the tangent cone from the satellite to the surface of the Earth.

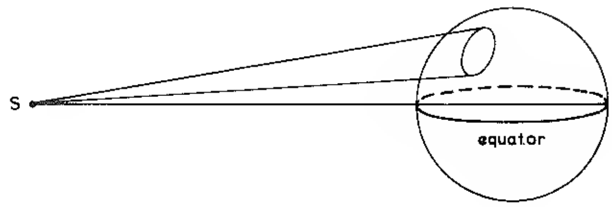

Fig. 1 - Illustration of a typical service area (for narrow beamwidth)

In general, it is more convenient to specify the boundary of the service area by plotting latitude against longitude. Fig. 2 (reproduced from reference 1) indicates the general nature of such curves.

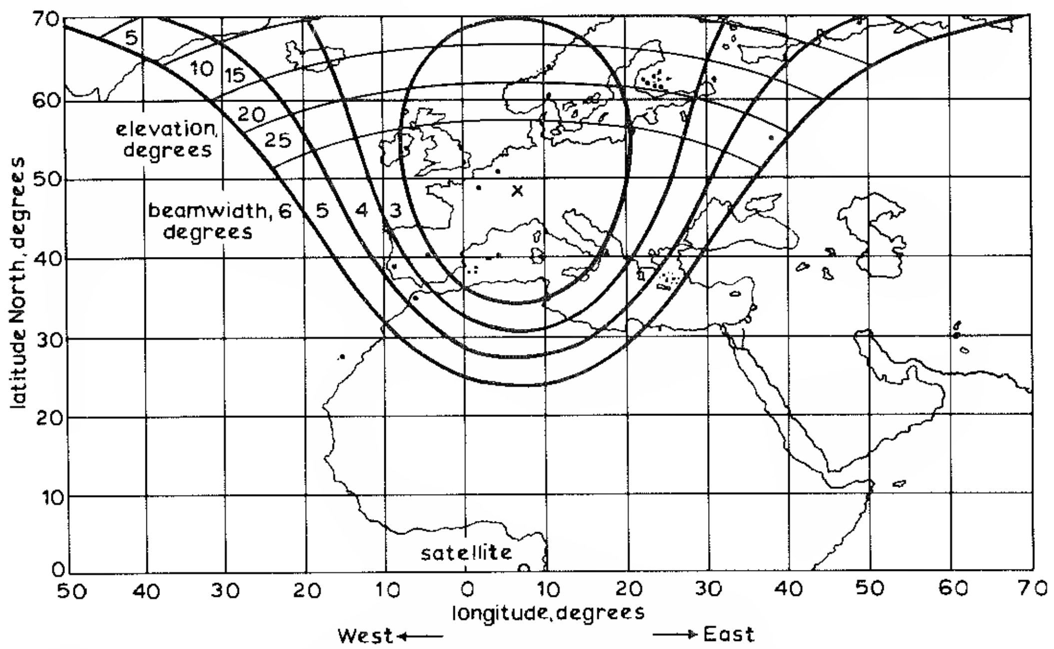

Fig. 2 - Coverage of Europe with the satellite at 8°E, Paris as the target point and varying beamwidth

The object of this report is to provide formulae by means of which the boundary of the service area can be determined whatever the beamwidth. The notation, based mainly upon the geometry of Fig. 3, is explained in Section 2. The case when the beamwidth is small is considered in Section 3; axial ratios and directions are tabulated for ten-degree intervals of the latitude of the target point, and of the difference between the longitudes of the target point and the subsatellite point.

If latitude is plotted against longitude, it can be shown that the small-beamwidth service-area boundaries remain ellipses, but as the beamwidth increases, they become non-elliptical closed curves until the cone of radiation from the satellite meets the tangent cone from the satellite to the Earth. For still greater beamwidth, we have the 'tulip' shape of some of the curves in Fig. 2. To determine a typical point of such a curve, it is necessary to find first the coordinates of a geometrically-specified point Q_n on the boundary of the service area and in the plane through the target point P_1 normal to the line SP_1 joining P_1 to the satellite S. Thereafter we have to determine whether the line SQ_n meets the Earth's surface, and, if so, the coordinates of P_n, the point of intersection. Formulae determining the coordinates of Q_n, and the coordinates of P_n in terms of those of Q_n, are given in Section 4. Section 6 gives conclusions.

2. NOTATION AND THE GEOMETRICAL CONSIDERATIONS UNDERLYING IT

Most of the points which are relevant to the determination of the service area are in the plane SCP_1 (the plane of the paper in Fig. 3), where S is the satellite, P_1 the target point and C the centre of the Earth. The coordinate axes (not shown in Fig. 3) are such that Cz is the axis of the Earth's rotation, and Cx is the intersection of the Equator and the Greenwich meridian; for a right-handed system of axes, latitude North, and longitude East of Greenwich, must be taken as positive. We shall use the following symbols:–

C	The centre of the Earth (and origin of coordinates)
D	The distance CS (taken as 26,300 miles (42,200 km) in numerical work)
K_1	The point on the Equator due South of P_1
L_1, M_1	Arbitrary points in the plane SCP_1 such that $L_1P_1M_1$ is the tangent to the Earth at P_1 in that plane.
P_1	The target point
P_n $(n \neq 1)$	A point where the line SQ_n meets the Earth's surface
Q_2, Q_3	Points in the plane SCP_1, in the plane through P_1 normal to SP_1 and on the boundary of the service area
Q_n $(n \neq 1,2,3)$	Any other point on the boundary of the service area and in the plane through P_1 normal to SP_1
r	The distance Q_nP_1 $(= SP_1 \tan\frac{1}{2}\alpha)$
R	The radius of the Earth (taken as 3960 miles (6375 km) in numerical work)
s	The subsatellite point
S	The position of the satellite (co-ordinates $(D\cos\phi_s, D\sin\phi_s, 0)$)
x_n	$(= R\cos\theta_n\cos\phi_n)$ x – coordinate of P_n
y_n	$(= R\cos\theta_n\sin\phi_n)$ y – coordinate of P_n
z_n	$(= R\sin\theta_n)$ z – coordinate of P_n
$x_1 + X_n$, $y_1 + Y_n$, $z_1 + Z_n$	Coordinates of Q_n
α	The beamwidth
γ_n	The angle between CP_1 produced and P_1Q_n
θ_n	The latitude of P_n
ξ_n	The angle $Q_nP_1Q_2$
ϕ_n	The longitude of P_n
ϕ_s	The longitude of s
ψ_1	The angle SCP_1
χ_1	The elevation of S as seen from P_1
ω_1	The bearing (West or East of South) of s from P_1

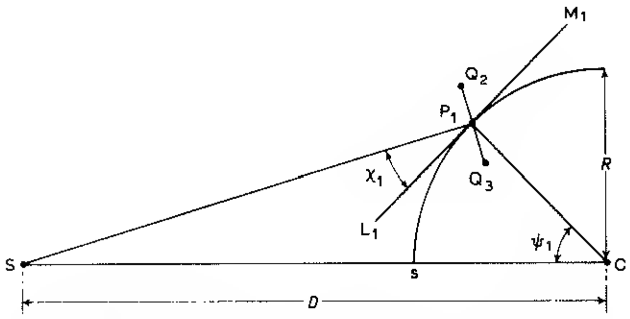

Fig. 3 - Relevant points associated with the plane formed by the satellite, the target point and the centre of the Earth

3. AXIAL LENGTHS AND DIRECTIONS FOR THE NARROW-BEAM CASE

From the geometry of Fig. 3 the following relations can be established:–

$$\sin\chi_1 = \frac{D\cos\psi_1 - R}{SP_1} \; ; \; \cos\chi_1 = D\sin\psi_1/SP_1 \quad (1)$$

$$\cos\gamma_n = \cos\chi_1 \cos\xi_n = D\sin\psi_1 \cos\xi_n/SP_1 \quad (2)$$

$$SP_1^{\,2} = D^2 + R^2 - 2DR\cos\psi_1 \quad (3)$$

$$\cos\psi_1 = \cos\theta_1 \cos(\phi_1 - \phi_s) \quad (4)$$

and from the spherical triangle P_1sK_1, right-angled at K_1

$$\sin\omega_1 = \sin(\phi_s - \phi_1)/\sin\psi_1 \quad (5)$$

For the narrow-beam case, we can regard the beam from S as a right-circular cylinder of radius r, cut by the tangent plane to the Earth (in which the elliptical service-area boundary lies) at an angle $(\pi/2 - \chi_1)$ with the plane through P_1 normal to SP_1. Hence the major axis of the ellipse is in the direction $L_1P_1M_1$ and has length $r\operatorname{cosec}\chi_1$, and the minor axis (in the direction through P_1 perpendicular to the plane SCP_1) has length r. The ratio $\operatorname{cosec}\chi_1$ of the axial lengths can be deduced directly from the first of (1), and is tabulated in Table 1, while the direction of the major axis is determined by ω_1 in Equation (5) and is tabulated in Table 2.

TABLE 1

Axial Ratio

$\phi_s - \phi_1$ \ θ_1	0°	10°	20°	30°	40°	50°	60°	70°	80°
0°	1	1·021	1·090	1·220	1·446	1·850	2·673	5·009	42·68
10°	1·021	1·043	1·113	1·246	1·477	1·891	2·736	5·153	48·21
20°	1·090	1·113	1·188	1·330	1·577	2·022	2·940	5·633	78·25
30°	1·220	1·246	1·330	1·489	1·769	2·277	3·344	6·634	–
40°	1·446	1·477	1·577	1·769	2·108	2·736	4·098	8·718	–
50°	1·850	1·891	2·022	2·277	2·736	3·609	5·633	14·12	–
60°	2·673	2·736	2·940	3·344	4·098	5·633	9·789	48·21	–
70°	5·009	5·153	5·633	6·634	8·718	14·12	48·21	–	–
80°	42·68	48·21	78·25	–	–	–	–	–	–

TABLE 2

Direction of Major Axis
Angle (ω_1) from a North-South Line is Tabulated

$\phi_s - \phi_1$ \ θ_1	0°	10°	20°	30°	40°	50°	60°	70°	80°
0°	–	0°	0°	0°	0°	0°	0°	0°	0°
10°	90°	45°27'	27°16½'	19°25½'	15°20'	12°58'	11°30½'	10°38'	10°9'
20°	90°	64°30'	46°47'	36°3'	29°31'	25°25'	22°48'	21°10'	20°17'
30°	90°	73°16'	59°22'	49°6'	41°56'	37°0'	33°41½'	31°34'	–
40°	90°	78°18½'	67°49'	59°13'	52°33'	47°36'	44°6'	41°46'	–
50°	90°	81°42½'	73°59'	67°14½'	61°39½'	57°16'	54°0'	51°45'	–
60°	90°	84°16'	78°50'	73°54'	69°38½'	66°9'	63°26'	61°31'	–
70°	90°	86°22½'	82°54'	79°41'	76°50'	74°25'	72°30'	–	–
80°	90°	88°14'	86°33'	–	–	–	–	–	–

4. DETERMINATION OF A TYPICAL POINT OF THE BOUNDARY OF THE SERVICE AREA IN THE GENERAL CASE

When the beamwidth is not necessarily small, we seek to determine the latitude θ_n and longitude ϕ_n of a typical point P_n of the boundary of the service area, and for this purpose it is convenient first to find the coordinates of the point Q_n in which SP_n meets the plane through P_1 normal to SP_1. We assume that Q_n is specified in terms of r (which is determined by the beamwidth and the position of the target point P_1, and is thus a known constant) and the angle ξ_n or $Q_nP_1Q_2$. Then since $P_1Q_n = r$

$$X_n^2 + Y_n^2 + Z_n^2 = r^2 \tag{6}$$

Since P_1Q_n is perpendicular to SP_1,

$$(D\cos\phi_s - R\cos\theta_1\cos\phi_1)X_n + (D\sin\phi_s - R\cos\theta_1\sin\phi_1)Y_n - R\sin\theta_1 Z_n = 0 \tag{7}$$

and since the angle between CP_1 produced and P_1Q_n is γ_n specified by equation (2), it can also be shown that

$$\cos\theta_1\cos\phi_1 X_n + \cos\theta_1\sin\phi_1 Y_n + \sin\theta_1 Z_n = rD\sin\psi_1\cos\xi_n/SP_1 \tag{8}$$

In general, (that is, if $\phi_1 \neq \phi_s$) (6), (7) and (8) can be reduced to

$$Z_n^2\sin^2\psi_1 - 2rZ_n\cos\xi_n\sin\theta_1\sin\psi_1\left[D - R\cos\psi_1\right]/SP_1 + r^2\left\{\sin^2\theta_1 - \sin^2\psi_1\sin^2\xi_n - (R^2\sin^2\theta_1\sin^2\psi_1\cos^2\xi_n/SP_1^2)\right\} = 0 \tag{9}$$

and X_n, Y_n are then obtained from

$$\left.\begin{aligned} X_n\sin(\phi_1 - \phi_s) &= B\sin\phi_1 - A\sin\phi_s \\ Y_n\sin(\phi_1 - \phi_s) &= A\cos\phi_s - B\cos\phi_1 \end{aligned}\right\} \tag{10}$$

where

$$\left.\begin{aligned} A &= \left\{(rD\sin\psi_1\cos\xi_n)/SP_1 - Z_n\sin\theta_1\right\}/\cos\theta_1 \\ B &= (rR\sin\psi_1\cos\xi_n)/SP_1 \end{aligned}\right\} \tag{11}$$

Given r, ξ_n [as well as θ_1, $(\phi_1 - \phi_s)$ and hence ψ_1] (9) gives two values of Z_n as a multiple of r, and then (10) and (11) give the corresponding values of X_n and Y_n as multiples of r. Equations (9), (10) and (11) are unaltered if ξ_n is replaced by $-\xi_n$: this explains why a quadratic equation for Z_n is to be expected. If $\phi_1 = \phi_s$, the corresponding solution is explicit, namely

$$\left.\begin{aligned} X_n &= r\left[\pm\sin\phi_1\sin\xi_n + (R/SP_1)\sin\theta_1\cos\phi_1\cos\xi_n\right] \\ Y_n &= r\left[\mp\cos\phi_1\sin\xi_n + (R/SP_1)\sin\theta_1\sin\phi_1\cos\xi_n\right] \\ Z_n &= r\cos\xi_n(D - R\cos\theta_1)/SP_1 \end{aligned}\right\} \tag{12}$$

At this stage we are in a position to determine explicitly the coordinates of Q_n, which are

$$(x_1 + X_n,\ y_1 + Y_n,\ z_1 + Z_n),$$

when r and ξ_n are given; it remains to determine the latitude and longitude of the point P_n in which SQ_n meets the Earth's surface. It is sufficient to determine the coordinates (x_n, y_n, z_n) of P_n; it can be shown that these satisfy the equations

$$\frac{x_n - x_1 - X_n}{D\cos\phi_s - x_1 - X_n} = \frac{y_n - y_1 - Y_n}{D\sin\phi_s - y_1 - Y_n} = \frac{z_1 + Z_n - z_n}{z_1 + Z_n} = \eta, \text{ say} \tag{13}$$

where

$$\eta^2\left[SP_1^2 + r^2 - (2rRD\sin\psi_1\cos\xi_n/SP_1)\right] - 2\eta\left[DR\cos\psi_1 + (rRD\sin\psi_1\cos\xi_n/SP_1) - R^2 - r^2\right] + r^2 = 0 \tag{14}$$

Equation (14) (of which only the smaller root is relevant) only involves ξ_n, r and known quantities, so η can be determined without knowing the position of Q_n explicitly. Once η is known, the last of equations (13) gives z_n and hence θ_n directly. Either of the remaining equations (13) then gives x_n or y_n and hence ϕ_n.

We are therefore now able to plot the latitude θ_n against the longitude ϕ_n in the general case, using a linear scale for both quantities. The geographical significance of the results is best appreciated by appropriately distorting the map of the world in the relevant region, since it is only major geographical features with which we are concerned.

If the beamwidth α (and therefore r) is sufficiently small, the line SQ_2 in Fig. 3 will meet the Earth's surface at a point P_2 (not shown in Fig. 3) on the great circle sP_1 produced, but if the beamwidth is too large, the line SQ_2 will fail to meet the Earth. The critical situation occurs when the line SQ_2P_2 touches the Earth at P_2, so that the angle P_2SC is $\sin^{-1}(R/D)$ or 8°40'. If angle P_2SC exceeds this value, there must be a value of ξ_n such that angle P_nSC is

$$\sin^{-1}(R/D)$$

instead, and the point P_n represents the theoretical extremity of the 'tulip' in such a case. At this point P_n, the elevation of the satellite is zero, so that service, though theoretically possible, will be poor. In practice, the service area should be regarded as terminated when the elevation of the satellite has a minimum value of at least a few degrees: the 'tulip'-shaped curves of Fig. 2 are for this reason terminated before the theoretical extremity is reached.

5. MODIFICATION FOR A CONICAL BEAM OF ELLIPTICAL CROSS-SECTION

If the beam emitted from the satellite is an arbitrary cone, meeting the plane through P_1 normal to SP_1 in an ellipse with centre P_1, the only difference will be that the distance r to a point on the service-area boundary in the plane through P_1 normal to SP_1 will no longer be constant and equal to $SP_1 \tan \frac{1}{2}\alpha$, but will instead be given by a formula of the form

$$\frac{1}{r^2} = \frac{1}{r_1^2}\cos^2(\xi_n - \xi_0) + \frac{1}{r_2^2}\sin^2(\xi_n - \xi_0) \qquad (15)$$

where $r_1 = SP_1 \tan \frac{1}{2}\alpha_1$ is the distance to a point of the service-area boundary in the direction of maximum beamwidth $\frac{1}{2}\alpha_1$, ξ_0 is the value of ξ_n in this direction, and $r_2 = SP_1 \tan \frac{1}{2}\alpha_2$ is the distance to a point of the service-area boundary in the perpendicular direction of minimum beamwidth $\frac{1}{2}\alpha_2$. In equation (15) there are three distinct essential parameters, r_1, r_2 and ξ_0, whereas when $r_1 = r_2$ the value of ξ_0 is immaterial and we only have one parameter.

Qualitively, we can expect the same general features as before: nearly elliptical service areas for sufficiently small beamwidth, and 'tulip'-shaped service areas for larger beamwidth. But as the orientation of the beam now has to be considered, there is no longer necessarily symmetry of the service area boundaries with respect to the plane SP_1C of Fig. 1, and, as already noted, there are three significant parameters to consider instead of only one. No detailed quantitative investigation will therefore be undertaken here, although all the mathematical machinery for such an investigation is available in the formulae already derived if a practical requirement should arise.

6. CONCLUSIONS

For a right-circular beam of sufficiently small beamwidth, the service-area boundary is approximately an ellipse in the tangent plane to the Earth, centred at the point towards which the beam is directed. The axial lengths and directions of this ellipse are determined by simple formulae based on the geometry of Fig. 3. For larger beamwidths the service area boundaries have the 'tulip' shape so noticeable in Fig. 2 (taken from Reference 1); formulae are given for determining the position of an arbitrary point of such a service area, and in particular, for determining the theoretical extremity of the 'tulip'. The case of a conical beam which is not right-circular is qualitatively similar and could be fully investigated by means of the formulae given here, but has not been attempted because it is qualitatively much more complicated, since three significant parameters are involved instead of one.

7. REFERENCE

1. BENOÎT, A., GODFROID, H. *and* KNYPERS, P., 1968. Distribution of television by satellite, with special reference to the size of the Earth-station aerials. *E.B.U. Rev.*, 1968, Part A, No. 110, pp. 162–172.

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SATELLITE BROADCASTING SERVICE AREAS

The author regrets that he has recently found out that Equation (14) of the original text is wrong, and should read

$$\eta^2(\mathrm{SP}_1^2 + r^2) + \eta[(D^2 - \mathrm{SP}_1^2) - (r^2 + \mathrm{CQ}_n{}^2)] + \mathrm{CQ}_n{}^2 - R^2 = 0$$

where

$$\mathrm{CQ}_n{}^2 = R^2 + r^2 + (2rRD \sin\psi_1 \cos\xi_n)/\mathrm{SP}_1$$

It may be useful to note that Equation (9) also has an explicit solution

$$Z_n = r\sin\theta_1 \cos\xi_n[D - R\cos\psi_1]/(\mathrm{SP}_1 \sin\psi_1) \pm r\cos\theta_1 \sin\xi_n \sin(\phi_1 - \phi_s)/\sin\psi_1$$

and this can be used to simplify the formulae (10) for X_n and Y_n in a form that covers all cases.

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